

ON THE THEORETICAL ANALYSIS OF THE SMALL x DATA IN RECENT HERA PAPERS**F. J. Ynduráin**

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Abstract

A number of internal inconsistencies, misleading conclusions and lack of completeness in the analyses of the small x deep inelastic data in recent papers of HERA physicists, particularly in a very recent one by the H1 collaboration are pointed out. It is also shown that, when an analysis without prejudice is carried over, the situation is very different from the one claimed there in what respects checks of theoretical expectations.

The so-called H1 Collaboration has recently produced a preprint concerning deep inelastic data at small x , that we will henceforth call HERA2. The work constitutes an improvement of the former HERA analysis^[1] of the structure function F_2 , to be denoted by HERA1, extending it to wider x , Q^2 ranges and greatly diminishing the errors.

There is not much to be argued about the *experimental* obtaining of the data. The machine and the detectors worked wonderfully. The quality of the data is excellent. It is therefore more of a pity that the physicists in the collaboration have not been more careful with the subsequent theoretical analysis which shows features that, to put it as mildly as possible, are difficult to justify. Because the second paper HERA2 (ref.2) is so much superior in the quality of the data, I will mostly discuss it here. Regarding it, I will address myself to a number of separate (but related) questions which I will now enumerate: **1) Parametrizations of the data.** **2) Behaviour of structure functions as $x \rightarrow 0$.** **3) Theoretical analysis with so-called “Double scaling limit”.** **4) Connection with various theoretical models.** This is the central part of the present note, which will then be finished with some comments.

1.-The theory of strong interactions is QCD. On this practically everybody is agreed, so, when presenting parametrizations of data one should try to, at least, not violate grossly standard QCD features. In this, I think, it is no excuse that the parametrization is merely “a phenomenological ansatz”, as is described the parametrization of p. 20 of HERA2, Eq. (8). As should be by now widely known, the structure function F_2 may be split in a singlet and a nonsinglet piece, $F_2 = F_S + F_{NS}$. For small x , $F_{NS} \simeq B_{NS}x^{0.5}$. *No matter what*, the fact that both F_S and F_{NS} are proportional to cross sections implies that $B_{NS} > 0$: “phenomenological ansatz” or not. A measure of the quality of the fit is obtained by noting that the corresponding coefficient in (8), denoted by e in the table underneath this Eq. (8) in HERA2 is *negative*. Really.

But there is more. It may perhaps be argued that Eq.(8) was not meant to have anything to do with a QCD description: at least it should be compatible with the rest of the paper HERA2. In the same page 20, the authors state that the wide range of Q^2 covered by their experiment allows them to study the behaviour at small x , and its variation with Q^2 . So they write

$$F_2(x, Q^2) \underset{x \rightarrow 0}{\simeq} x^{-\lambda} \quad (1)$$

and ascribe this prediction to De Rújula et al.^[3] We will come to this last point later; for now what interests us is that the HERA2 collaboration find, and report in Table 4, values of λ increasing as*

$$\lambda = 0.24 \ (Q^2 = 12 \text{ GeV}^2) \text{ to } \lambda = 0.50 \ (Q^2 = 800 \text{ GeV}^2). \quad (2)$$

Actually, a careful look at Table 4 (p.23) shows that, at least up to $Q^2 = 350 \text{ GeV}^2$, λ may be considered constant modulo what may easily be interpreted as statistical fluctuations, with a value of λ around $\lambda \sim 0.32$. Again, we will return to this later; for now what interests me is the blatant contradiction between Eq.(2) and the parametrization of Eq.(8) where, for $x \rightarrow 0$, one has $F_2 \simeq c x^d$ with $d = -0.188$, widely off the bounds given in Eq.(2). How the authors of HERA2 may claim that their data are fitted, *at the same time* by $x^{-0.32}$ and $x^{-0.19}$ baffles me. But there is more.

2.-It is extremely important, for judging the quality of the theoretical analysis of HERA2, to return to the matter of behaviour at small x . The authors of HERA2 fit F_2 at small x ; and they are so certain of their results that they display them not only in the text, but in Table 4 (to which I have already referred). The *smallest* value of λ they find is for $Q^2 = 2.5 \text{ GeV}^2$, where they get $\lambda = 0.19$; for $Q^2 = 1.5 \text{ GeV}^2$ the value is 0.21. In fact, an analysis at $Q^2 = 0$ (Compton scattering) gives^[4] a λ around 0.26.

Then the authors of HERA2 state that “the rate of growth of F_2 is expected to increase”, and promptly signal to ref.3 as the place where such growth was predicted. So one opens ref. 3, and there one reads the prediction (p. 1651, line 9ff): “The rate of growth increases with increasing Q^2 . *It is always weaker than a power*” of x , although stronger than a log. (Italics mine). So, the HERA people who both in their phenomenological parametrization [Eq. (8) of HERA2] and in their Table 4 find *a power*, conclude that they check the prediction of ref. 3 –a surprising conclusion.

3.-To clarify matters a bit more, I would like to bring attention to the derivation of the results of ref.3. Specifically, in p.1650 these authors consider the moments of the structure functions. They realize [their Eq. (2)] that the moments are given in terms of the Wilson expansion which, after a slight change of their rather old fashioned notation to the one prevalent nowadays, implies the relation

$$\mu_n(Q^2) \simeq \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{D(n)} a_n \simeq \left[\frac{\log Q^2}{\log Q_0^2} \right]^{D(n)} a_n. \quad (3)$$

The a_n may easily be identified with expectation values of operators,

$$a_n \sim \langle p | O_n | p \rangle$$

that need not be specified here. The $D(n)$ are related to the anomalous dimensions. De Rújula et al. recognize that the behaviour of F_2 for small x is linked to the singularities (in n , considered as a continuous variable) of the moments; the leading behaviour being given by the *rightmost* singularity. This is clear since one has

$$\mu_n(Q^2) = \int_0^1 dx F_2(x, Q^2) x^{n-2}.$$

Now, if one *assumes* that at a certain Q_0^2 one has a Pomeron-type behaviour,

$$F_2(x, Q_0^2) \underset{x \rightarrow 0}{\simeq} C_0, C_0 = \text{constant}, \quad (4)$$

then one has, at that Q_0^2 , a singularity for $n = 1$ that one can identify with that of the anomalous dimension, $D(n)$, which indeed is singular at $n = 1$.

It should be emphasized that here one has a dichotomy. If at a given Q_0^2 one has a behaviour like a constant, then the structure function, by virtue of the analysis of De Rújula et al.^[3] will grow “always weaker than a power” of x , in their own words. Contrarywise, if at *any* Q^2 , $F_2(x, Q^2)$ grows like a power,

* We only report the values from $Q^2 = 12 \text{ GeV}^2$; below this value the contamination of nonsinglet is very strong, and even rough analyses should incorporate this fact to get any credibility.

the conditions of the theorem cannot apply. Actually, the situation is really very simple. As discovered long ago^[6], one has *two* possibilities. Either the singularities of the matrix elements a_n lie to the left of those of $D(n)$, i.e., to the left of $n = 1$, or to the right of it. If the first, one has the situation envisaged in ref.3. If, however, the dominating singularity of a_n lies *to the right* of $n = 1$, at a certain $n = 1 + \lambda$, $\lambda > 0$, then one has a behaviour like a *power*, for all Q^2 , $F_2(x, Q^2) \simeq x^{-\lambda}$. The coefficient of proportionality, a function of Q^2 , may in fact be calculated. One may describe this result by stating that in the low x regime, the dependence on x and Q^2 of structure functions “factorizes”.

The paper of De Rújula et al. is not very explicit. To find a more detailed calculation, we will refer to the review of Ball and Forte^[5], apparently known to the HERA2 collaboration since they quote it. There, in p.9, the authors explicitly state that, if $F_2 \sim x^{-\lambda}$, the chain of arguments which lead to a “double scaling form”, namely their Eq.(2.31), breaks down, and a power-like behaviour becomes dominant [their Eq.(2.34), when one undoes their changes of variable, reads exactly $F_2(x, Q^2) \simeq x^{-\lambda}$]. Curiously enough, this power behaviour is exactly what is found by both HERA collaborations.

We can be more specific. Assume, with De Rújula et al, and Ball and Forte, that, contrary to the findings of the HERA2 group, one had, at a given Q_0^2 , a Pomeron-like behaviour as that given by Eq.(4) in the present paper. Then, as correctly stated in refs.3, 5, one obtains a behaviour dominated by the (known) singularities of $D(n)$. A simple calculation then gives, for all $Q^2 \gg Q_0^2$, $x \rightarrow 0$, the very explicit formula

$$F_2(x, Q^2) \underset{x \rightarrow 0}{\underset{Q^2 \rightarrow \infty}{\simeq}} C_0 \left[\frac{33 - 2n_f}{576\pi^2 |\log x| \log[\alpha_s(Q_0^2)/\alpha_s(Q^2)]} \right]^{\frac{1}{4}} \exp \sqrt{\frac{144 |\log x|}{(33 - 2n_f)} \left[\log \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]}. \quad (5)$$

This equation is essentially identical to the corresponding one in ref.5, p.9; the behaviour it implies has been described as “double asymptotic scaling”. Roughly (but we will come to more precise statements below) Eq. (5) states that

$$F_2(x, Q^2) \simeq \exp \sqrt{|\log x| f_1}. \quad (6)$$

On the other hand, and in the same notation, the findings of the HERA2 fits, in exponential form, imply

$$F_2(x, Q^2) \simeq \exp\{|\log x| f_2\}, \quad (7)$$

where the f_i may depend weakly on Q^2 . Undaunted by the evident incompatibility of (6) and (7), the authors in HERA2 happily assert at the same time the validity of (7), in their Table 4, and Eq. (8), and the validity of (6) in p. 26: “Thus double asymptotic scaling is a dominant feature in this region”.

4.-How can these contradictory statements be reconciled? They cannot. The physicists in HERA2 have failed, in their theoretical analysis of the data, to do what an honest experimentalist should do, *viz.* to test theoretical assumptions without bias.

In fact there existed at the time of the first and (of course) the second HERA analyses *three* theoretical predictions (that I know of). From a standard moments analysis one can either have the “double asymptotic scaling”, refs.3, 5:

$$F_2(x, Q^2) \underset{x \rightarrow 0}{\underset{Q^2 \rightarrow \infty}{\simeq}} C_0 \left[\frac{33 - 2n_f}{576\pi^2 |\log x| \log[\alpha_s(Q_0^2)/\alpha_s(Q^2)]} \right]^{\frac{1}{4}} \exp \sqrt{\frac{144 |\log x|}{(33 - 2n_f)} \left[\log \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]} + B_{NS}[\alpha_s(Q^2)]^{-D_{11}} x^{0.5}. \quad (8)$$

This depends on the singularity of $D(n)$ being the rightmost one. On the other hand, if it is the singularity of a_n that lies to the right, one gets the factorization behaviour given in ref.6:

$$F_2(x, Q^2) \simeq B_S[\alpha_s(Q^2)]^{-d_+} x^{-\lambda} + B_{NS}[\alpha_s(Q^2)]^{-D_{11}} x^{0.5}; \quad (9)$$

I have added the nonsinglet contribution $B_{NS}[\alpha_s(Q^2)]^{-D_{11}} x^{0.5}$, $D_{11} \simeq 0.512$, to both expressions for future ease of reference. In (9), d_+ is the largest eigenvalue of D , easily calculated; an explicit formula for it may

be found in refs.4 or 6. It is to be stressed that, in the present state of the art, one cannot decide on the basis of perturbative QCD alone* on which is valid, (8) or (9): it should be for experimentalists to tell us.

There is yet a *third* theoretical estimate^[7]. It is very difficult to compare with the other two, as it is obtained using totally different methods to get the prediction

$$F_2(x, Q^2) \simeq x^{-\omega_0 \alpha_s}, \quad \omega_0 = \frac{4C_A \log 2}{\pi}, \quad C_A = 3. \quad (10)$$

Let me finish this point by telling what one gets if trying to fit (8), (9) or (10) to the HERA data. This test will be done in two steps. I will choose only data with $x < 10^{-2}$, to be sure that one is in the “small x ” region and, in the first fits, I will also take $100 \text{ GeV}^2 > Q_0^2 > 10 \text{ GeV}^2$ to avoid problems with varying number of excited flavours, large NLO corrections, and to have, for the test of (8), $Q^2 \gg Q_0^2$, whatever the last may be. This region contains a total of 48 high-quality points. In the second step, the set of points will be enlarged to Q^2 up to 350 GeV^2 . We will then have 63 experimental points, distributed over a wide range in Q^2 . Thus, the analysis will necessitate inclusion of NLO corrections. To the approximation needed, these may be implemented by simply taking $\alpha_s(Q^2)$ to two loops, and allowing Λ to vary freely, which is what I will do here.

The results are now summarized, starting with the first situation. (10) runs contrary to the trend of the data, and produces a very large chi-squared, $\chi^2/\text{d.o.f.} = 926/(48 - 2)$. What is more, the fit rejects the introduction of a NS component, a clear indication of the fact that the behaviour (10) is certainly not attained at the HERA energies (the fit to the higher Q^2 data does not fare much better, either). Thus we will say no more about (10); the interested reader may find further discussion, and a possible re-interpretation of (10) in ref.4.

For the fits with Eqs.(8), (9), we fix $n_f = 4$, take α_s to one loop,

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \log Q^2/\Lambda},$$

and choose $\Lambda = 0.2 \text{ GeV}$ so that $\alpha_s(m_\tau^2) = 0.32$. The results would not change much qualitatively if allowing, e.g., Λ to be a free parameter (some such results may be found in ref.4, and in the present note below).

The fit with the formula of De Rújula et al., *not* taking into account the NS piece is quite bad, but the results improve when including a NS contribution, as is done in Eq. (8). They are summarized in Table A1.

C_0	B_{NS}	Q_0^2	$\chi^2/\text{d.o.f.}$
7.1×10^{-3}	2.0	0.10 GeV^2	$51.3/(48 - 3)$

Table A1.- Fit with “double scaling”, Eq.(8). $\Lambda = 0.2$ (fixed)

This should, of course, be compared with the fit one gets (Table B1) if using the equation (9), since (9) and (8) stem from mutually incompatible assumptions. If we neglected the NS contribution in (9), we would also get a poor chi-squared; when including the NS piece this improves to about one unit by d.o.f., see Table B1.

B_S	B_{NS}	λ	$\chi^2/\text{d.o.f.}$
1.04×10^{-2}	1.21	0.40	$49.8/(48 - 3)$

Table B1.- Fit with “factorization”, Eq.(9). $\Lambda = 0.2$ (fixed)

The results reported in Table B1 are more satisfactory than those given in Table A1: not so much the chi-squared by d.o.f., only marginally better, but because of the following other features. Firstly, the value of B_{NS} in Table B1 is closer to what one obtains in the analysis of structure functions like νW_3 in neutrino scattering, which are pure nonsinglet and which suggest $B_{NS} \sim 0.6$. Another problem with the results reported in Table A1 is that they imply that the structure function at Q_0^2 , essentially proportional to the Compton scattering cross section, should behave as a constant with the energy (proportional to $1/x$), which it does *not*: as shown in refs.4, 8, 9, it still grows like a power of $1/x$.

* For deep inelastic γp scattering. For $\gamma^* \gamma$ the situation is different, and will be discussed below.

The situation is even more clear if we use the second set of HERA2 data with $x > 10^{-2}$, and all Q^2 between 12 and 350 GeV². Here one has to take into account NLO corrections, which, as explained, may be well approximated by just using the two-loop formula for $\alpha_s(Q^2)$, leaving Λ as a free parameter.

For the “double scaling” hypothesis, Eq.(8), the resulting chi-squared/d.o.f. *deteriorates*. Numerically, I have conducted a fit with (8) and α_s to two loops and find the results of Table A2.

Λ	C_0	B_{NS}	Q_0^2	$\chi^2/\text{d.o.f.}$
0.40 GeV	4.9×10^{-3}	2.0	0.38 GeV ²	69.9/(63 – 4)

Table A2.- Fit with “double scaling”, Eq.(8); Λ taken as a free parameter. $n_f = 4$.

That (8) does not give a very brilliant fit may also be seen in Fig.10 of HERA2 (p.25), particularly in the lower part. In spite of the use of a log log plot, on which anything looks like a straight line, eyeball inspection of the scattering of the experimental points around the theoretical curve indicates a chi-squared/d.o.f. clearly larger than unity (the authors of HERA2 seem to be allergic to numbers, and do not give any figure for this or several other chi-squared of their fits). On top of it, the two problems mentioned in connection with the former fit, non-Pomeron behaviour at $Q_0^2 = 0.38 \text{ GeV}^2$ and deviation of B_{NS} from its expected value persist.

For Eq.(9), the details of the analysis may be found in ref.4; the chi-squared per d.o.f. actually *improves* to less than one unit, and B_{NS} decreases to the value of 0.8, quite close to the neutrino result of 0.6. This is summarized in Table B2.

Λ	B_0	B_{NS}	λ	$\chi^2/\text{d.o.f.}$
0.10 GeV	1.0×10^{-3}	0.77	0.38	50.4/(63 – 4)

Table B2.- Fit with “factorization”, Eq.(9); Λ taken as a free parameter. $n_f = 4$.

The $\chi^2/\text{d.o.f.}$ is now substantially better in the fit with (9) than in the fit with (8), 50.4 vs. 69.9; but other features also improve. Thus the value of B_{NS} in Table B2 is now comfortably close to that expected from neutrino deep inelastic scattering, 0.77 compared to 0.6.

5.- Comments. One can of course still not draw definite conclusions, but there is a clear trend: the data of HERA, both the old HERA1 and the new HERA2, support clearly a “factorization” behaviour at small x , as predicted on the basis of the analysis of lower energy data long ago by López and the present author^[6]. As shown in the present note the “double scaling” description, which the paper HERA2 claims to provide evidence for* (“double asymptotic scaling is a dominant feature”) actually fails to give a good fit to the data, besides presenting extra problems. It is really sad that the HERA1 and HERA2 collaborations have not taken the trouble to present a fair analysis of experiment, not such a difficult task as I have been shown here. Why they ignore the possibility of a power-like behaviour as that given in Eq.(9) is really difficult to understand; apart from the original papers, it is given in the textbook of ref.10 considered (by e.g. the Particle Data Group) as one of the standard references in QCD. The situation is even more outrageous since many of the physicists involved in HERA1 and HERA2 have also been involved in the PETRA analyses of deep inelastic $\gamma^*\gamma$ scattering for which Witten^[11] has derived (and experiment has checked) a behaviour, at small x , *exactly* like that in (9), with a value of λ , $\lambda \sim 0.5$ constant up to a slight dependence in the number of flavours excited. This was indeed one of the theoretical reasons given in the original papers^[6] for preferring (9) to the “double scaling” behaviour.

* In all fairness it should be noted that the authors of HERA2 admit the existence of some deviations of the data from the predictions of double asymptotic scaling, e.g., in the last line of p.24.

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